FFTPACK5 DOCUMENTATION

FFTPACK 5.1 -- a FORTRAN library of fast Fourier transforms

Complex Transform Routines

- CFFT1I 1D complex initialization
- CFFT1B 1D complex backward
- CFFT1F 1D complex forward
- CFFT2I 2D complex initialization
- CFFT2B 2D complex backward
- CFFT2F 2D complex forward
- CFFTMI multiple complex initialization
- CFFTMB multiple complex backward
- CFFTMF multiple complex forward

Real Transform Routines

- RFFT1I 1D real initialization
- RFFT1B 1D real backward
- RFFT1F 1D real forward
- RFFT2I 2D real initialization
- RFFT2B 2D real backward
- RFFT2F 2D real forward
- RFFTMI multiple real initialization
- RFFTMB multiple real backward
- RFFTMF multiple real forward

Real Cosine Transform Routines

- COST1I 1D real cosine initialization
- COST1B 1D real cosine backward
- COST1F 1D real cosine forward
- COSTMI multiple real cosine initialization
- COSTMB multiple real cosine backward
- COSTMF multiple real cosine forward

Real Sine Transform Routines

- SINT1I 1D real sine initialization
- SINT1B 1D real sine backward
- SINT1F 1D real sine forward
- SINTMI multiple real sine initialization
- SINTMB multiple real sine backward
- SINTMF multiple real sine forward

Real Quarter-Cosine Transform Routines

- COSQ1I 1D real quarter-cosine initialization
- COSQ1B 1D real quarter-cosine backward
- COSQ1F 1D real quarter-cosine forward
- COSQMI multiple real quarter-cosine initialization
- COSQMB multiple real quarter-cosine backward
- COSQMF multiple real quarter-cosine forward

Real Quarter-Sine Transform Routines

- SINQ1I 1D real quarter-sine initialization
- SINQ1B 1D real quarter-sine backward
- SINQ1F 1D real quarter-sine forward
- SINQMI multiple real quarter-sine initialization
- SINQMB multiple real quarter-sine backward
- SINQMF multiple real quarter-sine forward

CFFT1I - initialization routine for CFFT1B and CFFT1F

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SYNOPSIS

```
 SUBROUTINE CFFT1I (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine CFFT1I initializes
array WSAVE for use in

 its companion routines CFFT1B and CFFT1F.
Routine CFFT1I must
```

```
 be called before the first call to CFFT1B or
CFFT1F, and after

 whenever the value of integer N changes.


 Input Arguments


 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least

         2*N + INT(LOG(REAL(N))/LOG(2.)) + 4.




 Output Arguments


 WSAVE   Real work array with dimension LENSAV,
containing the

         prime factors of N and also containing
certain trigonometric

         values which will be used in routines
CFFT1B or CFFT1F.
```

```
 IER     = 0 successful exit

         = 2 input parameter LENSAV not big
enough
```

CFFT1B - complex backward fast Fourier transform

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SYNOPSIS

```
 SUBROUTINE CFFT1B (N, INC, C, LENC, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENC, LENSAV, LENWRK, IER

 COMPLEX    C(LENC)

 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine CFFT1B computes the one-
dimensional Fourier
```

```
 transform of a single periodic sequence within
a complex array.

 This transform is referred to as the backward
transform or Fourier

 synthesis, transforming the sequence from
spectral to physical

 space.



 This transform is normalized since a call to
CFFT1B followed

 by a call to CFFT1F (or vice-versa) reproduces
the original

 array subject to algorithm constraints,
roundoff error, etc.



 Input Arguments



 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.



 INC     Integer increment between the
locations, in array C, of two
```

consecutive elements within the sequence to be transformed.

C Complex array of length LENC containing the sequence to be

transformed.

LENC Integer dimension of C array. LENC must be at least

INC*(N-1) + 1.

WSAVE Real work array with dimension LENSAV. WSAVE's contents

must be initialized with a call to subroutine CFFT1I before

the first call to routine CFFT1F or CFFT1B for a given

transform length N. WSAVE's contents may be re-used for

subsequent calls to CFFT1F and CFFT1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

```
          2*N + INT(LOG(REAL(N))/LOG(2.)) + 4.



 WORK     Real work array of dimension LENWRK.


 LENWRK   Integer dimension of WORK array.
LENWRK must be at

          least 2*N.



 Output Arguments


 C        For index J*INC+1 where J=0,...,N-1,


             C(J*INC+1) =


             N-1
             SUM C(K*INC+1)*EXP(I*J*K*2*PI/N)
             K=0


          where I=SQRT(-1).
```

```
         At other indices, the output value of
C does not differ

         from input.


 IER     =  0 successful exit

         =  1 input parameter LENC   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         = 20 input error returned by lower
level routine
```

CFFT1F - complex forward fast Fourier transform

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SYNOPSIS

```
 SUBROUTINE CFFT1F (N, INC, C, LENC, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENC, LENSAV, LENWRK, IER
```

```
 COMPLEX    C(LENC)

 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine CFFT1F computes the one-
dimensional Fourier

 transform of a single periodic sequence within
a complex array.

 This transform is referred to as the forward
transform or Fourier

 analysis, transforming the sequence from
physical to spectral

 space.


 This transform is normalized since a call to
CFFT1F followed

 by a call to CFFT1B (or vice-versa) reproduces
the original

 array subject to algorithm constraints,
roundoff error, etc.


 Input Arguments


 N       Integer length of the sequence to be
transformed.  The
```

transform is most efficient when N is a product of

small primes.

INC Integer increment between the locations, in array C, of two

consecutive elements within the sequence to be transformed.

C Complex array of length LENC containing the sequence to be

transformed.

LENC Integer dimension of C array. LENC must be at least

INC*(N-1) + 1.

WSAVE Real work array with dimension LENSAV. WSAVE's contents

must be initialized with a call to subroutine CFFT1I before

the first call to routine CFFT1F or CFFT1B for a given

transform length N. WSAVE's contents may be re-used for

subsequent calls to CFFT1F and CFFT1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG(REAL(N))/LOG(2.)) + 4.

WORK Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least 2*N.

Output Arguments

C For index J*INC+1 where J=0,...,N-1 (that is, for the Jth

element of the sequence),

C(J*INC+1) =

N-1

SUM C(K*INC+1)*EXP(-I*J*K*2*PI/N)

K=0

```
          where I=SQRT(-1).


          At other indices, the output value of
C does not differ
          from input.


 IER      =  0 successful exit
          =  1 input parameter LENC   not big
enough
          =  2 input parameter LENSAV not big
enough
          =  3 input parameter LENWRK not big
enough
          = 20 input error returned by lower
level routine
```

CFFT2I - initialization routine for CFFT2B, CFFT2F

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SYNOPSIS

```
 SUBROUTINE CFFT2I (L, M, WSAVE, LENSAV, IER)
```

```
 INTEGER    L, M, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 routine CFFT2I initializes real
array WSAVE for use

 in its companion routines CFFT2F and CFFT2B
for computing two-

 dimensional fast Fourier transforms of complex
data.  Prime

 factorizations of L and M, together with
tabulations of the

 trigonometric functions, are computed and
stored in array WSAVE.


 Input Arguments


 L       Integer number of elements to be
transformed in the first

         dimension.  The transform is most
efficient when L is a

         product of small primes.
```

M Integer number of elements to be transformed in the second

dimension. The transform is most efficient when M is a

product of small primes.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*(L+M) + INT(LOG(REAL(L))/LOG(2.)) + INT(LOG(REAL(M))/LOG(2.)) + 8.

Output Arguments

WSAVE Real work array with dimension LENSAV, containing the

prime factors of L and M, and also containing certain

trigonometric values which will be used in routines

CFFT2B or CFFT2F.

WSAVE Real work array with dimension LENSAV. The WSAVE array

```
         must be initialized with a call to
subroutine CFFT2I before

         the first call to CFFT2B or CFFT2F,
and thereafter whenever

         the values of L, M or the contents of
array WSAVE change.

         Using different WSAVE arrays for
different transform lengths

         or types in the same program may
reduce computation costs

         because the array contents can be re-
used.




 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough

         = 20 input error returned by lower
level routine
```

CFFT2B - complex, two-dimensional backward fast Fourier transform

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SYNOPSIS

```
 SUBROUTINE CFFT2B (LDIM, L, M, C, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    L, M, LDIM, LENSAV, LENWRK, IER

 COMPLEX    C(LDIM,M)

 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine CFFT2B computes the two-
dimensional discrete

 Fourier transform of a complex periodic array.
This transform is

 known as the backward transform or Fourier
synthesis, transforming

 from spectral to physical space.



 Routine CFFT2B is normalized, in that a call
to CFFT2B followed

 by a call to CFFT2F (or vice-versa) reproduces
the original array

 subject to algorithm constraints, roundoff
error, etc.
```

Input Arguments

LDIM Integer first dimension of two-dimensional complex array C.

L Integer number of elements to be transformed in the first

dimension of the two-dimensional complex array C. The value

of L must be less than or equal to that of LDIM. The

transform is most efficient when L is a product of small

primes.

M Integer number of elements to be transformed in the second

dimension of the two-dimensional complex array C. The

transform is most efficient when M is a product of small

primes.

C Complex array of two dimensions containing the (L,M) subarray

to be transformed. C's first dimension is LDIM, its second

dimension must be at least M.

WSAVE Real work array with dimension LENSAV. WSAVE's contents

must be initialized with a call to subroutine CFFT2I before

the first call to routine CFFT2F or CFFT2B with transform

lengths L and M. WSAVE's contents may be re-used for

subsequent calls to CFFT2F and CFFT2B with the same

transform lengths L and M.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*(L+M) + INT(LOG(REAL(L))/LOG(2.)) + INT(LOG(REAL(M))/LOG(2.)) + 8.

WORK Real work array.

LENWRK Integer dimension of WORK array. LENWRK must be at least

2*L*M.

Output Arguments

C Complex output array. For purposes of exposition,

assume the index ranges of array C are defined by

C(0:L-1,0:M-1).

For I=0,...,L-1 and J=0,...,M-1, the C(I,J)'s are given

in the traditional aliased form by

$$C(I,J) = \sum_{L1=0}^{L-1} \sum_{M1=0}^{M-1} C(L1,M1) * EXP(SQRT(-1)*2*PI*(I*L1/L + J*M1/M))$$

And in unaliased form, the C(I,J)'s are given by

```
                    LF    MF
          C(I,J) = SUM   SUM   C(L1,M1,K1)*
                   L1=LS M1=MS

                      EXP(SQRT(-1)*2*PI*(I*L1/L
+ J*M1/M))

          where

             LS= -L/2    and LF=L/2-1   if L is
even;
             LS=-(L-1)/2 and LF=(L-1)/2 if L is
odd;
             MS= -M/2    and MF=M/2-1   if M is
even;
             MS=-(M-1)/2 and MF=(M-1)/2 if M is
odd;

          and

             C(L1,M1) = C(L1+L,M1) if L1 is
zero or negative;
             C(L1,M1) = C(L1,M1+M) if M1 is
zero or negative;
```

```
          The two forms give different results
when used to

          interpolate between elements of the
sequence.


  IER     Integer error return

          =  0 successful exit

          =  2 input parameter LENSAV not big
enough

          =  3 input parameter LENWRK not big
enough

          =  5 input parameter L > LDIM

          = 20 input error returned by lower
level routine
```

CFFT2F - complex, two-dimensional forward fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE CFFT2F (LDIM, L, M, C, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    L, M, LDIM, LENSAV, LENWRK, IER

 COMPLEX    C(LDIM,M)

 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine CFFT2F computes the two-
dimensional discrete

 Fourier transform of a complex periodic array.
This transform is

 known as the forward transform or Fourier
analysis, transforming

 from physical to spectral space.



 Routine CFFT2F is normalized, in that a call
to CFFT2F followed

 by a call to CFFT2B (or vice-versa) reproduces
the original array

 subject to algorithm constraints, roundoff
error, etc.




 Input Arguments
```

LDIM Integer first dimension of two-dimensional complex array C.

L Integer number of elements to be transformed in the first

dimension of the two-dimensional complex array C. The value

of L must be less than or equal to that of LDIM. The

transform is most efficient when L is a product of small

primes.

M Integer number of elements to be transformed in the second

dimension of the two-dimensional complex array C. The

transform is most efficient when M is a product of small

primes.

C Complex array of two dimensions containing the (L,M) subarray

to be transformed. C's first dimension is LDIM, its second

dimension must be at least M.

WSAVE Real work array with dimension LENSAV. WSAVE's contents

must be initialized with a call to subroutine CFFT2I before

the first call to routine CFFT2F or CFFT2B with transform

lengths L and M. WSAVE's contents may be re-used for

subsequent calls to CFFT2F and CFFT2B having those same

transform lengths.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*(L+M) + INT(LOG(REAL(L))/LOG(2.)) + INT(LOG(REAL(M))/LOG(2.)) + 8.

WORK Real work array.

LENWRK Integer dimension of WORK array. LENWRK must be at least

2*L*M.

Output Arguments

C Complex output array. For purposes of exposition,

assume the index ranges of array C are defined by

C(0:L-1,0:M-1).

For I=0,...,L-1 and J=0,...,M-1, the C(I,J)'s are given

in the traditional aliased form by

$$C(I,J) = 1/(L*M)*\sum_{L1=0}^{L-1}\sum_{M1=0}^{M-1} C(L1,M1)*$$

```
                    EXP(-SQRT(-1)*2*PI*(I*L1/L
+ J*M1/M))


         And in unaliased form, the C(I,J)'s
are given by


                              LF    MF

         C(I,J) =  1/(L*M)*SUM   SUM
C(L1,M1)*

                              L1=LS M1=MS


                    EXP(-SQRT(-1)*2*PI*(I*L1/L
+ J*M1/M))


         where


            LS= -L/2    and LF=L/2-1   if L is
even;

            LS=-(L-1)/2 and LF=(L-1)/2 if L is
odd;

            MS= -M/2    and MF=M/2-1   if M is
even;

            MS=-(M-1)/2 and MF=(M-1)/2 if M is
odd;


         and
```

```
            C(L1,M1) = C(L1+L,M1) if L1 is zero
or negative;

            C(L1,M1) = C(L1,M1+M) if M1 is zero
or negative;


         The two forms give different results
when used to

         interpolate between elements of the
sequence.


 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         =  5 input parameter L > LDIM

         = 20 input error returned by lower
level routine
```

CFFTMI - initialization routine for CFFTMB and CFFTMF

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C     *                                   by
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C     *                Paul Swarztrauber and
Dick Valent                *
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SYNOPSIS

```
 SUBROUTINE CFFTMI (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine CFFTMI initializes
array WSAVE for use in

 its companion routines CFFTMB and CFFTMF.
Routine CFFTMI must

 be called before the first call to  CFFTMB or
CFFTMF, and after

 whenever the value of integer N changes.


 Input Arguments


 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.
```

```
 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least

         2*N + INT(LOG(REAL(N))/LOG(2.)) + 4.



 Output Arguments


 WSAVE   Real work array with dimension LENSAV,
containing the

         prime factors of N and also containing
certain trigonometric

         values which will be used in routines
CFFTMB or CFFTMF.



 IER     = 0 successful exit

         = 2 input parameter LENSAV not big
enough
```

CFFTMB - complex, multiple backward fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE CFFTMB (LOT, JUMP, N, INC, C, LENC,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENC, LENSAV,
LENWRK, IER

 COMPLEX    C(LENC)

 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine CFFTMB computes the one-
dimensional Fourier

 transform of multiple periodic sequences
within a complex array.

 This transform is referred to as the backward
transform or Fourier

 synthesis, transforming the sequences from
spectral to physical

 space.


 This transform is normalized since a call to
CFFTMF followed
```

by a call to CFFTMB (or vice-versa) reproduces the original

array subject to algorithmic constraints, roundoff error, etc.

Input Arguments

LOT Integer number of sequences to be transformed within

array C.

JUMP Integer increment between the locations, in array C,

of the first elements of two consecutive sequences

to be transformed.

N Integer length of each sequence to be transformed. The

transform is most efficient when N is a product of

small primes.

INC Integer increment between the locations, in array C,

```
         of two consecutive elements within the
same sequence

         to be transformed.


 C       Complex array containing LOT
sequences, each having

         length N, to be transformed.  C can
have any number

         of dimensions, but the total number of
locations must

         be at least LENC.


 LENC    Integer dimension of C array.  LENC
must be at

         least (LOT-1)*JUMP + INC*(N-1) + 1.


 WSAVE   Real work array of length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine CFFTMI before the

         first call to routine CFFTMF or CFFTMB
for a given transform

         length N.

 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least

         2*N + INT(LOG(REAL(N))/LOG(2.)) + 4.
```

WORK Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least 2*LOT*N.

Output Arguments

C For index L*JUMP+J*INC+1 where J=0,...,N-1 and

L=0,...,LOT-1, (that is, for the Jth element of the Lth

sequence),

C(L*JUMP+J*INC+1) =

N-1

SUM C(L*JUMP+K*INC+1)*EXP(I*J*K*2*PI/N)

K=0

```
          where I=SQRT(-1).


          At other indices, the output value of
C does not differ

          from input.


 IER      = 0 successful exit

          = 1 input parameter LENC   not big
enough

          = 2 input parameter LENSAV not big
enough

          = 3 input parameter LENWRK not big
enough

          = 4 input parameters INC,JUMP,N,LOT
are not consistent.


              The parameters integers INC, JUMP,
N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.


              For multiple FFTs to execute
correctly, input variables
```

```
              INC, JUMP, N and LOT must be
consistent ... otherwise at

              least one array element mistakenly
is transformed more

              than once.
```

CFFTMF - complex, multiple forward fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE CFFTMF (LOT, JUMP, N, INC, C, LENC,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENC, LENSAV,
LENWRK, IER

 COMPLEX    C(LENC)

 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine CFFTMF computes the one-
dimensional Fourier
```

```
 transform of multiple periodic sequences
within a complex array.

 This transform is referred to as the forward
transform or Fourier

 analysis, transforming the sequences from
physical to spectral

 space.



 This transform is normalized since a call to
CFFTMF followed

 by a call to CFFTMB (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.



 Input Arguments



 LOT     Integer number of sequences to be
transformed within

         array C.



 JUMP    Integer increment between the
locations, in array C,

         of the first elements of two
consecutive sequences

         to be transformed.
```

```
 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 INC     Integer increment between the
locations, in array C,

         of two consecutive elements within the
same sequence

         to be transformed.


 C       Complex array containing LOT
sequences, each having

         length N, to be transformed.  C can
have any number

         of dimensions, but the total number of
locations must

         be at least LENC.


 LENC    Integer dimension of C array.  LENC
must be at

         least (LOT-1)*JUMP + INC*(N-1) + 1.
```

```
 WSAVE   Real work array of length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine CFFTMI before the

         first call to routine CFFTMF or CFFTMB
for a given transform

         length N.

 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least

         2*N + INT(LOG(REAL(N))/LOG(2.)) + 4.




 WORK    Real work array of dimension LENWRK.


 LENWRK  Integer dimension of WORK array.
LENWRK must be at

         least 2*LOT*N.




 Output Arguments


 C       For index L*JUMP + J*INC +1 where
J=0,...,N-1 and

         L=0,...,LOT-1, (that is, for the Jth
element of the Lth
```

```
          sequence),


             C(L*JUMP+J*INC+1) =


             N-1

             SUM C(L*JUMP+K*INC+1)*EXP(-
I*J*K*2*PI/N)

             K=0


          where I=SQRT(-1).


          At other indices, the output value of
C does not differ

          from input.


 IER      = 0 successful exit

          = 1 input parameter LENC   not big
enough

          = 2 input parameter LENSAV not big
enough

          = 3 input parameter LENWRK not big
enough

          = 4 input parameters INC,JUMP,N,LOT
are not consistent.
```

```
              The parameters integers INC, JUMP,
N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.



              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent ... otherwise at

              least one array element mistakenly
is transformed more

              than once.
```

RFFT1I - initialization routine for RFFT1B and RFFT1F

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```

SYNOPSIS

```
 SUBROUTINE RFFT1I (N, WSAVE, LENSAV, IER)
```

```
 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine RFFT1I initializes
array WSAVE for use

 in its companion routines RFFT1B and RFFT1F.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate

 WSAVE arrays are required for different values
of N.


 Input Arguments


 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least
```

```
          N + INT(LOG (REAL(N))/LOG(2.)) +4.



 Output Arguments


 WSAVE   Real work array with dimension LENSAV,
containing the

         prime factors of N and also containing
certain trigonometric

         values which will be used in routines
RFFT1B or RFFT1F.



 IER     =  0 successful exit

         =  2 input parameter LENSAV not big
enough
```

RFFT1B - real backward fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE RFFT1B (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine RFFT1B computes the one-
dimensional Fourier

 transform of a periodic sequence within a real
array.  This

 is referred to as the backward transform or
Fourier synthesis,

 transforming the sequence from spectral to
physical space.


 This transform is normalized since a call to
RFFT1B followed

 by a call to RFFT1F (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.


 Input Arguments
```

```
 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
sequence.


 R       Real array of length LENR containing
the sequence to be

         transformed.


 LENR    Integer dimension of R array.  LENR
must be at least

         INC*(N-1) + 1.


 WSAVE   Real work array o length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine RFFT1I before the

         first call to routine RFFT1F or RFFT1B
for a given transform

         length N.
```

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(0:(N-1)*INC).

The output values of R are written over the input values.

If N is even, set NH=N/2-1; then for J=0,...,N-1

```
          R(J*INC) =     R(0) +


              [(-1)**J*R((N-1)*INC)]


                       NH

                +      SUM  R((2*N1-
1)*INC)*COS(J*N1*2*PI/N)

                       N1=1


                       NH

                +      SUM
R(2*N1*INC)*SIN(J*N1*2*PI/N)

                       N1=1
```

If N is odd, set NH=(N-1)/2 and define R as above,

except remove the expression in square brackets [].

IER	Integer error return
	= 0 successful exit
	= 1 input parameter LENR not big enough

```
          =  2 input parameter LENSAV not big
enough

          =  3 input parameter LENWRK not big
enough


```

RFFT1F - real backward fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE RFFT1F (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine RFFT1F computes the one-
dimensional Fourier

 transform of a periodic sequence within a real
array.  This

 is referred to as the forward transform or
Fourier analysis,
```

```
 transforming the sequence from physical to
spectral space.


 This transform is normalized since a call to
RFFT1F followed

 by a call to RFFT1B (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.


 Input Arguments


 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
sequence.


 R       Real array of length LENR containing
the sequence to be

         transformed.
```

LENR Integer dimension of R array. LENR must be at least

INC*(N-1) + 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine RFFT1I before the

first call to routine RFFT1F or RFFT1B for a given transform

length N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(0:(N-1)*INC).

Then

$$R(0) = \sum_{N1=0}^{N-1} R(N1*INC)/N$$

If N is even, set NH=N/2-1; if N is odd set NH=(N-1)/2;

then for J=1,...,NH

$$R((2*J-1)*INC) = \sum_{N1=0}^{N-1} 2.*R(N1*INC)*\cos(J*N1*2*PI/N)/N$$

```
          and


            R(2*J*INC) =


                        N-1

                   2.*SUM
(R(N1*INC)*SIN(J*N1*2*PI/N)/N

                     N1=0


          Also if N is even then


            R((N-1)*INC) =


                        N-1

                        SUM  (-1)**N1*R(N1*INC)/N

                        N1=0



 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough
```

```
          =  2 input parameter LENSAV not big
enough

          =  3 input parameter LENWRK not big
enough


```

RFFT2I - initialization routine for RFFT2B and RFFT2F

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```

SYNOPSIS

```
 SUBROUTINE RFFT2I (L, M, WSAVE, LENSAV, IER)

 INTEGER    L, M, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 routine RFFT2I initializes real
array WSAVE for use

 in its companion routines RFFT2F and RFFT2B
for computing the two-

 dimensional fast Fourier transform of real
data.  Prime

 factorizations of L and M, together with
tabulations of the
```

```
 trigonometric functions, are computed and
stored in array WSAVE.

 RFFT2I must be called prior to the first call
to RFFT2F or RFFT2B.

 Separate WSAVE arrays are required for
different values of L or M.


 Input Arguments


 L       Integer number of elements to be
transformed in the first

         dimension.  The transform is most
efficient when L is a

         product of small primes.


 M       Integer number of elements to be
transformed in the second

         dimension.  The transform is most
efficient when M is a

         product of small primes.


 LENSAV  Integer number of elements in the
WSAVE array.  LENSAV must

         be at least L + 3*M +
INT(LOG(REAL(L))/LOG(2.)) +

         2*INT(LOG(REAL(M))/LOG(2.)) +12.
```

```
 Output Arguments


 WSAVE   Real work array with dimension LENSAV,
containing the

         prime factors of L and M, and also
containing certain

         trigonometric values which will be
used in routines

         RFFT2B or RFFT2F.




 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough

         = 20 input error returned by lower
level routine
```

RFFT2B - complex to real, two-dimensional backward fast Fourier transform

```
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C     *                Paul Swarztrauber and
Dick Valent                *
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SYNOPSIS

```
 SUBROUTINE RFFT2B (LDIM, L, M, R, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    LDIM, L, M, LENSAV, LENWRK, IER

 REAL       R(LDIM,M), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine RFFT2B computes the two-
dimensional discrete

 Fourier transform of the complex Fourier
coefficients a real

 periodic array.  This transform is known as
the backward transform

 or Fourier synthesis, transforming from
spectral to physical space.



 Routine RFFT2B is normalized: a call to RFFT2B
followed by a

 call to RFFT2F (or vice-versa) reproduces the
original array

 subject to algorithmic sonstraints, roundoff
error, etc.
```

Input Arguments

LDIM Integer first dimension of the two-dimensional real

array R, which must be at least L.

L Integer number of elements to be transformed in the first

dimension of the two-dimensional real array R. The value

of L must be less than or equal to that of LDIM. The

transform is most efficient when L is a product of small

primes.

M Integer number of elements to be transformed in the second

dimension of the two-dimensional real array R. The

transform is most efficient and accurate when M is a product

of small primes.

```
 R       A real L by M array containing the
spectral coefficients

         of a real L by M array that are stored
as described in the

         documentation of subroutine rfft2f.
THe first dimension is

         LDIM which must be at least L. The
second dimension must be

         at least M.




 WSAVE   Real work array of length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine RFFT2I before the

         first call to routine RFFT2F or
RFFT2B. WSAVE's contents may

         be re-used for subsequent calls to
RFFT2F and RFFT2B with

         the same L and M.




 LENSAV  Integer number of elements in the
WSAVE array.  LENSAV must

         be at least L + 3*M +
INT(LOG(REAL(L))/LOG(2.)) +
```

2*INT(LOG(REAL(M))/LOG(2.)) +12.

WORK Real array of dimension LENWRK, where LENWRK is defined

below. WORK provides workspace, and its contents need not

be saved between calls to routines RFFT2B and RFFT2F.

LENWRK Integer number of elements in the WORK array. LENWRK must

be at least (L+1)*M.

Output Arguments

R A real L by M array. If the full transform c is reconstructed

using subroutine r2c(ldim,lcdim,l,m,r,c) then for i=0,...,l-1

and j=0,...,m-1

$$R(I,J) = \sum_{L1=0}^{L-1} \sum_{M1=0}^{M-1} C(L1,M1)$$

L1=0 M1=0

*EXP(SQRT(-1)*2*PI*(I*L1/L+J*M1/M))

or using the conjugate symmetry c(i,j)=c(l-1,m-j) this can

be written in terms of c(i,j), i=0,...,(L+1)/2

and j=0,...,m as:

M-1

R(I,J) = REAL[SUM C(0,M1)*EXP(SQRT(-1)*2*PI*J*M1/M]

M1=0

(L+1)/2-1 M-1

+ 2*REAL[SUM SUM C(L1,M1)*

L1=1 M1=0

```
                              *EXP(SQRT(-
1)*2*PI*(I*L1/L+J*M1/M)) ]


             If L is even then add


                          M-1

             + REAL[ SUM (-
1)**I*C(L/2,M1)*EXP(SQRT(-1)*2*PI*J*M1/M) ]

                          M1=0


             c(i,j) = a(i,j)+i*b(I,J) are
contained in the real output

             array r(i,j) except for c(0,m-j)
and c(l,m-j) which can

             be obtained from c(0,m-j) = c(0,j)
and

             c(l,j) = c(l.m-j) = c(l,j)




 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough
```

```
         =  3 input parameter LENWRK not big
enough

         =  6 input parameter LDIM is less than
2*INT((L+1)/2)

         = 20 input error returned by lower
level routine
```

RFFT2F - real to complex, two-dimensional forward fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE RFFT2F (LDIM, L, M, R, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    LDIM, L, M,  LENSAV, LENWRK, IER

 REAL       R(LDIM,M), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

FFTPACK 5.1 routine RFFT2F computes the two-dimensional discrete

Fourier transform of a real periodic array. This transform is

known as the forward transform or Fourier analysis, transforming

from physical to spectral space.

Routine RFFT2F is normalized: a call to RFFT2F followed by a

call to RFFT2B (or vice-versa) reproduces the original array

subject to algorithmic constraints, roundoff error, etc.

Input Arguments

LDIM Integer first dimension of the two-dimensional real

array R, which must be at least L.

L Integer number of elements to be transformed in the first

dimension of the two-dimensional real array R. The value

of L must be less than or equal to that of LDIM. The

transform is most efficient when L is a product of small

primes.

M Integer number of elements to be transformed in the second

dimension of the two-dimensional real array R. The

transform is most efficient when M is a product of small

primes.

R Real array of two dimensions containing the L-by-M subarray

to be transformed. R's first dimension is LDIM and its

second dimension must be at least as large as M.

WSAVE Real work array of length LENSAV. WSAVE's contents must

```
         be initialized with a call to
subroutine RFFT2I before the

         first call to routine RFFT2F or
RFFT2B. WSAVE's contents

         may be re-used for subsequent calls to
RFFT2F and RFFT2B

         as long as L and M remain unchanged.




 LENSAV  Integer number of elements in the
WSAVE array.  LENSAV must

         be at least L + 3*M +
INT(LOG(REAL(L))/LOG(2.)) +

         2*INT(LOG(REAL(M))/LOG(2.)) +12.




 WORK    Real array of dimension LENWRK which
is defined below.

         WORK provides workspace, and its
contents need not be saved

         between calls to routines RFFT2F and
RFFT2B.


 LENWRK  Integer number of elements in the WORK
array.  LENWRK must

         be at least (L+1)*M.
```

Output Arguments

R Real output array of two dimensions. The full complex transform

of r(i,j) is given by:

$$C(I,J) = 1/(L*M)*\sum_{L1=0}^{L-1}\sum_{M1=0}^{M-1} R(L1,M1)* EXP(-SQRT(-1)*2*PI*(I*L1/L + J*M1/M))$$

The complex transform of a real array has conjugate symmetry.

That is, c(i,j) = congugate c(l-i,m-j) so only half the transform

is computed and packed back into the original array R.

Examples: Let a(i,j) = re[c(i,j)] and b(i,j) = Im[c(i,j)] then

following the forward transform

For l=m=4

a(0,0) a(0,1) b(0,1) a(0,2)

r(i,j) = a(1,0) a(1,1) a(1,2) a(1,3)

b(1,0) b(1,1) b(1,2) b(1,3)

a(2,0) a(2,1) b(2,1) a(2,2)

For l=m=5

a(0,0) a(0,1) b(0,1) a(0,2),b(0,2)

a(1,0) a(1,1) a(1,2) a(1,3),a(1,4)

r(i,j) = b(1,0) b(1,1) b(1,2) b(1,3),b(1,4)

a(2,0) a(2,1) a(2,2) a(2,3),a(2,4)

b(2,0) b(2,1) b(2,2) b(2,3),b(2,4)

The remaining c(i,j) for i=int((L+1)/2)+1,..,L and m=0,...m-1

can be obtained via the conjugate symmetry, which also implies

```
         that c(0,j) = conjugate c(0,m-j) and
for even l,

         c(l/2,0) = conjugate c(l/2,m-j).


         The full complex transform c(i,j),
i=1,...,L and j=1,...,M can also

         be extracted using


                              subroutine
r2c(ldim,lcdim,l,m,r,c)


         where lcdim is the first dimension of
the complex array c, which

         must be greater than or equal to l.




 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         =  6 input parameter LDIM is less than
2*INT((L+1)/2)

         = 20 input error returned by lower
level routine
```

RFFTMI - initialization routine for RFFTMB and RFFTMF

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```

SYNOPSIS

```
 SUBROUTINE RFFTMI (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine RFFTMI initializes
array WSAVE for use

 in its companion routines RFFTMB and RFFTMF.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate
```

WSAVE arrays are required for different values of N.

Input Arguments

N Integer length of each sequence to be transformed. The

transform is most efficient when N is a product of

small primes.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

N + INT(LOG (REAL(N))/LOG(2.)) +4.

Output Arguments

WSAVE Real work array with dimension LENSAV, containing the

prime factors of N and also containing certain trigonometric

values which will be used in routines RFFTMB or RFFTMF.

```
 IER     =  0 successful exit

         =  2 input parameter LENSAV not big
enough
```

RFFTMB - real, multiple backward fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE RFFTMB (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine RFFTMB computes the one-
dimensional Fourier
```

```
 transform of multiple periodic sequences
within a real array.

 This transform is referred to as the backward
transform or Fourier

 synthesis, transforming the sequences from
spectral to physical

 space.



 This transform is normalized since a call to
RFFTMB followed

 by a call to RFFTMF (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.



 Input Arguments



 LOT     Integer number of sequences to be
transformed within

         array R.



 JUMP    Integer increment between the
locations, in array R,

         of the first elements of two
consecutive sequences

         to be transformed.
```

```
 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
same sequence.


 R       Real array containing LOT sequences,
each having length N.

         R can have any number of dimensions,
but the total number

         of locations must be at least LENR.


 LENR    Integer dimension of R array.  LENR
must be at

         least (LOT-1)*JUMP + INC*(N-1) + 1.



 WSAVE   Real work array of length LENSAV.
WSAVE's contents must
```

be initialized with a call to subroutine RFFTMI before the

first call to routine RFFTMF or RFFTMB for a given transform

length N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(0:(LOT-1)*JUMP+(N-1)*INC).

The output values of R are written over the input values.

If N is even, set NH=N/2-1; then for I=0,...,LOT-1 and

J=0,...,N-1

$$R(I*JUMP+J*INC) = R(I*JUMP) +$$

$$[(-1)**J*R(I*JUMP+(N-1)*INC)]$$

$$+ \sum_{N1=1}^{NH} R(I*JUMP+(2*N1-1)*INC)*COS(J*N1*2*PI/N)$$

$$+ \sum_{N1=1}^{NH} R(I*JUMP+2*N1*INC)*SIN(J*N1*2*PI/N)$$

If N is odd, set NH=(N-1)/2 and define R as above,

```
          except remove the expression in
square brackets [].


 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         =  4 input parameters INC,JUMP,N,LOT
are not consistent.


              The parameters integers INC,
JUMP, N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.


              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent ... otherwise at
```

```
               least one array element
mistakenly is transformed more

               than once.
```

RFFTMF - real, multiple forward fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE RFFTMF (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine RFFTMF computes the one-
dimensional Fourier

 transform of multiple periodic sequences
within a real array.
```

```
 This transform is referred to as the forward
transform or Fourier

 analysis, transforming the sequences from
physical to spectral

 space.



 This transform is normalized since a call to
RFFTMF followed

 by a call to RFFTMB (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.



 Input Arguments



 LOT     Integer number of sequences to be
transformed within

         array R.



 JUMP    Integer increment between the
locations, in array R,

         of the first elements of two
consecutive sequences

         to be transformed.
```

```
 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
same sequence.


 R       Real array containing LOT sequences,
each having length N.

         R can have any number of dimensions,
but the total number

         of locations must be at least LENR.


 LENR    Integer dimension of R array.  LENR
must be at

         least (LOT-1)*JUMP + INC*(N-1) + 1.




 WSAVE   Real work array o length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine RFFTMI before the
```

first call to routine RFFTMF or RFFTMB for a given transform

length N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(0:(LOT-1)*JUMP+(N-1)*INC).

Then for I=0,...,LOT-1

$$R(I*JUMP) = \sum_{N1=0}^{N-1} R(I*JUMP+N1*INC)/N$$

If N is even, set NH=N/2-1; if N is odd set NH=(N-1)/2;

then for J=1,...,NH

$$R(I*JUMP+(2*J-1)*INC) = 2.*\sum_{N1=0}^{N-1} (R(I*JUMP+N1*INC)*COS(J*N1*2*PI/N)/N$$

and

$$R(I*JUMP+2*J*INC) = 2.*\sum_{N1=0}^{N-1} (R(I*JUMP+N1*INC)*SIN(J*N1*2*PI/N)/N$$

N1=0

Also if N is even then

R(I*JUMP+(N-1)*INC) =

N-1

SUM (-1)**N1*R(I*JUMP+N1*INC)/N

N1=0

IER Integer error return

= 0 successful exit

= 1 input parameter LENR not big enough

= 2 input parameter LENSAV not big enough

= 3 input parameter LENWRK not big enough

= 4 input parameters INC,JUMP,N,LOT are not consistent.

The parameters integers INC, JUMP, N and LOT are

consistent if equality

```
              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.



              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent ... otherwise at

              least one array element
mistakenly is transformed more

              than once.
```

COST1I - initialization routine for COST1B and COST1F

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```

SYNOPSIS

```
 SUBROUTINE COST1I (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER
```

```
 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine COST1I initializes
array WSAVE for use

 in its companion routines COST1F and COST1B.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate

 WSAVE arrays are required for different values
of N.



 Input Arguments



 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N-1
is a product of

         small primes.



 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least

         2*N + INT(LOG (REAL(N))/LOG(2.)) +4.
```

```
 Output Arguments


 WSAVE   Real work array with dimension LENSAV,
containing the

         prime factors of N and also containing
certain trigonometric

         values which will be used in routines
COST1B or COST1F.




 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough

         = 20 input error returned by lower
level routine
```

COST1B - real backward cosine fast Fourier transform

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C     *                Paul Swarztrauber and
Dick Valent               *
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```

SYNOPSIS

```
 SUBROUTINE COST1B (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine COST1B computes the one-
dimensional Fourier

 transform of an even sequence within a real
array.  This

 transform is referred to as the backward
transform or Fourier

 synthesis, transforming the sequence from
spectral to physical

 space.



 This transform is normalized since a call to
COST1B followed

 by a call to COST1F (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.



 Input Arguments
```

```
 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N-1
is a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
sequence.


 R       Real array of length LENR containing
the sequence to be

         transformed.


 LENR    Integer dimension of R array.  LENR
must be at least

         INC*(N-1)+ 1.




 WSAVE   Real work array of length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine COST1I before the
```

first call to routine COST1F or COST1B for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to COST1F and COST1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least N-1.

Output Arguments

R Real output array R. For purposes of exposition,

```
         assume R's range of indices is given
by

         R(0:(N-1)*INC).


         The output values of R are written
over the input values.

         For J=0,...,N-1


          R(J*INC) =


                N-1

                SUM  R(N1*INC)*COS(J*N1*PI/(N-1))

                N1=0


 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         = 20 input error returned by lower
level routine
```

COST1F - real backward cosine fast Fourier transform

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```

SYNOPSIS

```
SUBROUTINE COST1F (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.1 routine COST1F computes the one-
dimensional Fourier

transform of an even sequence within a real
array.  This transform

is referred to as the forward transform or
Fourier analysis,

transforming the sequence from  physical to
spectral space.


This transform is normalized since a call to
COST1F followed by a
```

```
call to COST1B (or vice-versa) reproduces the
original array

subject to algorithmic constraints, roundoff
error, etc.


Input Arguments


N       Integer length of the sequence to be
transformed.  The transform

        is most efficient when N-1 is a product
of small primes.


INC     Integer increment between the
locations, in array R of two consecutive

        elements within the sequence.


R       Real array of length LENR containing
the sequence to be transformed.


LENR    Integer dimension of R array.  LENR
must be at least INC*(N-1)+ 1.


WSAVE   Real work array of length LENSAV.
WSAVE's contents must be initialized

        with a call to subroutine COST1I before
the first call to routine COST1F
```

or COST1B for a given transform length N. WSAVE's contents may be re-used

for subsequent calls to COST1F and COST1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least N-1.

Output Arguments

R Real output array R. For purposes of this exposition, assume R's range

of indices is given by R(0:(N-1)*INC) and that the input values of R

are denoted by X. The output values of R are written over the input

values X as follows:

CASE N=1

```
--------

R(0) = X(0)

CASE N>1
--------

For J=0

R(0) =

   0.5*X(0)/(N-1)

   N-2
 + SUM  R(N1*INC)/(N-1)
   N1=1

 + 0.5*X((N-1)*INC)/(N-1)

For J=1,...,N-2

 R(J*INC) =
```

```
            R(0)/(N-1)


            N-2
          + SUM  2.0*(X(N1*INC)*COS(J*N1*PI/(N-
1)))/(N-1)
            N1=1


          + ((-1)**J)*X((N-1)*INC)/(N-1)


         R((N-1)*INC) =


             0.5*X(0)/(N-1)


             N-2
           + SUM  R(N1*INC)*((-1)**N1)/(N-1)
             N1=1


           + 0.5*((-1)**(N-1))*X((N-1)*INC)/(N-
1)


IER     Integer error return
        =  0 successful exit
```

```
        =  1 input parameter LENR   not big
enough

        =  2 input parameter LENSAV not big
enough

        =  3 input parameter LENWRK not big
enough

        = 20 input error returned by lower
level routine
```

COSTMI - initialization routine for COSTMB and COSTMF

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```

SYNOPSIS

```
 SUBROUTINE COSTMI (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

FFTPACK 5.1 subroutine COSTMI initializes array WSAVE for use

in its companion routines COSTMF and COSTMB. The prime factor-

ization of N together with a tabulation of the trigonometric

functions are computed and stored in array WSAVE. Separate

WSAVE arrays are required for different values of N.

Input Arguments

N Integer length of each sequence to be transformed. The

transform is most efficient when N is a product of

small primes.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4

Output Arguments

```
 WSAVE   Real work array with dimension LENSAV,
containing the

         prime factors of N and also containing
certain trigonometric

         values which will be used in routines
COSTMB or COSTMF.




 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough

         = 20 input error returned by lower
level routine
```

COSTMB - real, multiple backward cosine fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE COSTMB (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine COSTMB computes the one-
dimensional Fourier

 transform of multiple even sequences within a
real array.  This

 transform is referred to as the backward
transform or Fourier

 synthesis, transforming the sequences from
spectral to physical

 space.


 This transform is normalized since a call to
COSTMB followed

 by a call to COSTMF (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.


 Input Arguments
```

```
 LOT     Integer number of sequences to be
transformed within

         array R.


 JUMP    Integer increment between the
locations, in array R,

         of the first elements of two
consecutive sequences

         to be transformed.


 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N-1
is a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
same sequence.


 R       Real array containing LOT sequences,
each having length N.

         R can have any number of dimensions,
but the total number
```

of locations must be at least LENR.

LENR Integer dimension of R array. LENR must be at least

(LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine COSTMI before the

first call to routine COSTMF or COSTMB for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to COSTMF and COSTMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*(N+1).

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(0:(LOT-1)*JUMP+(N-1)*INC).

The output values of R are written over the input values.

For I=0,...,LOT-1 and J=0,...,N-1

$$R(I*JUMP+J*INC) = \sum_{N1=0}^{N-1} R(I*JUMP+N1*INC)*\cos(J*N1*PI/(N-1))$$

```
 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         =  4 input parameters INC,JUMP,N,LOT
are not consistent.

         = 20 input error returned by lower
level routine


              The parameters integers INC,
JUMP, N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.


              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent, otherwise at
```

```
               least one array element
mistakenly is transformed more

               than once.
```

COSTMF - real, multiple forward cosine fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE COSTMF (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine COSTMF computes the one-
dimensional Fourier

 transform of multiple even sequences within a
real array.  This
```

transform is referred to as the forward transform or Fourier

analysis, transforming the sequences from physical to spectral

space.

This transform is normalized since a call to COSTMF followed

by a call to COSTMB (or vice-versa) reproduces the original

array subject to algorithmic constraints, roundoff error, etc.

Input Arguments

LOT Integer number of sequences to be transformed within

array R.

JUMP Integer increment between the locations, in array R,

of the first elements of two consecutive sequences

to be transformed.

N Integer length of each sequence to be transformed. The

transform is most efficient when N-1 is a product of

small primes.

INC Integer increment between the locations, in array R,

of two consecutive elements within the same sequence.

R Real array containing LOT sequences, each having length N.

R can have any number of dimensions, but the total number

of locations must be at least LENR.

LENR Integer dimension of R array. LENR must be at least

(LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine COSTMI before the

first call to routine COSTMF or COSTMB for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to COSTMF and COSTMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*(N+1).

Output Arguments

R Real output array R. For purposes of exposition,

```
          assume R's range of indices is given
by

          R(0:(LOT-1)*JUMP+(N-1)*INC).


          The output values of R are written
over the input values.


          For I=0,...,LOT-1

           R(I*JUMP) =


                0.5*X(I*JUMP)/(N-1)


                N-2
              + SUM  R(I*JUMP+*N1*INC)/(N-1)
                N1=1


              + 0.5*X(I*JUMP+(N-1)*INC)/(N-1)


          For I=0,...,LOT-1 and J=1,...,N-2

           R(I*JUMP+J*INC) =


                R(I*JUMP)/(N-1)
```

```
                N-2

              + SUM
2.0*(X(I*JUMP+*N1*INC)*COS(J*N1*PI/(N-1)))/(N-
1)

                N1=1



              + ((-1)**J)*X(I*JUMP+(N-1)*INC)/(N-
1)



          For I=0,...,LOT-1

           R(I*JUMP+(N-1)*INC) =



                0.5*X(I*JUMP)/(N-1)



                N-2

              + SUM  R(I*JUMP+*N1*INC)*((-
1)**N1)/(N-1)

                N1=1



              + 0.5*((-1)**(N-1))*X(I*JUMP+(N-
1)*INC)/(N-1)



 IER      Integer error return

          =  0 successful exit
```

```
         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         =  4 input parameters INC,JUMP,N,LOT
are not consistent.

         = 20 input error returned by lower
level routine



              The parameters integers INC,
JUMP, N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.



              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent, otherwise at

              least one array element
mistakenly is transformed more

              than once.
```

SINT1I - initialization routine for SINT1B and SINT1F

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```

SYNOPSIS

```
 SUBROUTINE SINT1I (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine SINT1I initializes
array WSAVE for use

 in its companion routines SINT1F and SINT1B.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate

 WSAVE arrays are required for different values
of N.


 Input Arguments
```

```
 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N+1
is a product of

         small primes.


 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least

         N/2 + N + INT(LOG (REAL(N))/LOG(2.))
+4.




 Output Arguments


 WSAVE   Real work array with dimension LENSAV,
containing the

         prime factors of N and also containing
certain trigonometric

         values which will be used in routines
SINT1B or SINT1F.




 IER     Integer error return

         =  0 successful exit
```

```
          =  2 input parameter LENSAV not big
enough

          = 20 input error returned by lower
level routine
```

SINT1B - real backward sine fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE SINT1B (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine SINT1B computes the one-
dimensional Fourier

 transform of an odd sequence within a real
array.  This transform
```

```
 is referred to as the backward transform or
Fourier synthesis,

 transforming the sequence from spectral to
physical space.


 This transform is normalized since a call to
SINT1B followed

 by a call to SINT1F (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.


 Input Arguments


 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N+1
is a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
sequence.
```

R Real array of length LENR containing the sequence to be

transformed.

LENR Integer dimension of R array. LENR must be at least

INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine SINT1I before the

first call to routine SINT1F or SINT1B for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to SINT1F and SINT1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

N/2 + N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. Must be at least 2*N+2.

Output Arguments

R Real output array. For purposes of exposition,

assume R's range of indices is given by

R(INC:N*INC).

The output values of R are written over the input values.

For J=1,...,N

R(J*INC) =

$$\sum_{N1=1}^{N} R(N1*INC)*SIN(J*N1*PI/(N+1))$$

```
 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         = 20 input error returned by lower
level routine
```

SINT1F - real forward sine fast Fourier transform

```
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```

SYNOPSIS

```
 SUBROUTINE SINT1F (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine SINT1F computes the one-
dimensional Fourier

 transform of an odd sequence within a real
array.  This transform

 is referred to as the forward transform or
Fourier analysis,

 transforming the sequence from physical to
spectral space.



 This transform is normalized since a call to
SINT1F followed

 by a call to SINT1B (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.



 Input Arguments



 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N+1
is a product of

         small primes.
```

INC Integer increment between the locations, in array R,

of two consecutive elements within the sequence.

R Real array of length LENR containing the sequence to be

transformed.

LENR Integer dimension of R array. LENR must be at least

INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine SINT1I before the

first call to routine SINT1F or SINT1B for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to SINT1F and SINT1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

N/2 + N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. Must be at least 2*N+2.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by R(INC:(N-1)*INC).

The output values of R are written over the input values.

For J=1,...,N

R(J*INC) =

```
                N

               SUM
2.*R(N1*INC)*SIN(J*N1*PI/(N+1))/(N+1)

               N1=1


 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         = 20 input error returned by lower
level routine
```

SINTMI - initialization routine for SINTMB and SINTMF

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Dick Valent                  *
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```

SYNOPSIS

```
 SUBROUTINE SINTMI (N, WSAVE, LENSAV, IER)
```

```
 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine SINTMI initializes
array WSAVE for use

 in its companion routines SINTMF and SINTMB.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate

 WSAVE arrays are required for different values
of N.


 Input Arguments


 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least
```

```
          N/2 + N + INT(LOG (REAL(N))/LOG(2.))
+4.



 Output Arguments


 WSAVE   Real work array with dimension LENSAV,
containing the

         prime factors of N and also containing
certain trigonometric

         values which will be used in routines
SINTMB or SINTMF.



 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough

         = 20 input error returned by lower
level routine
```

SINTMB - real, multiple backward sine fast Fourier transform

```
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```

SYNOPSIS

```
 SUBROUTINE SINTMB (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine SINTMB computes the one-
dimensional Fourier

 transform of multiple odd sequences within a
real array.  This

 transform is referred to as the backward
transform or Fourier

 synthesis, transforming the sequences from
spectral to physical

 space.


 This transform is normalized since a call to
SINTMB followed
```

by a call to SINTMF (or vice-versa) reproduces the original

array subject to algorithmic constraints, roundoff error, etc.

Input Arguments

LOT Integer number of sequences to be transformed within

array R.

JUMP Integer increment between the locations, in array R,

of the first elements of two consecutive sequences.

N Integer length of each sequence to be transformed. The

transform is most efficient when N+1 is a product of

small primes.

INC Integer increment between the locations, in array R,

of two consecutive elements within the same sequence.

```
 R       Real array containing LOT sequences,
each having length N.

         R can have any number of dimensions,
but the total number

         of locations must be at least LENR.


 LENR    Integer dimension of R array.  LENR
must be at least

         (LOT-1)*JUMP + INC*(N-1)+ 1.




 WSAVE   Real work array of length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine SINTMI before the

         first call to routine SINTMF or SINTMB
for a given transform

         length N.  WSAVE's contents may be re-
used for subsequent

         calls to SINTMF and SINTMB with the
same N.


 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least
```

```
          N/2 + N + INT(LOG (REAL(N))/LOG(2.))
+4.



 WORK    Real work array of dimension at least
LENWRK.


 LENWRK  Integer dimension of WORK array.
LENWRK must be at

          least LOT*(2*N+4).


 Output Arguments


  R      Real output array.  For purposes of
exposition,

          assume R's range of indices is given
by

          R(INC:(LOT-1)*JUMP+N*INC).


          The output values of R are written
over the input values.

          For I=0,...,LOT-1 and J=1,...,N


           R(I*JUMP+J*INC) =
```

```
               N

               SUM
R(I*JUMP+*N1*INC)*SIN(J*N1*PI/(N+1))

               N1=1


 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         =  4 input parameters INC,JUMP,N,LOT
are not consistent.

         = 20 input error returned by lower
level routine


               The parameters integers INC,
JUMP, N and LOT are

               consistent if equality

               I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

               and J1,J2 < LOT implies I1=I2 and
J1=J2.
```

```
              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent ... otherwise at

              least one array element
mistakenly is transformed more

              than once.
```

SINTMF - real, multiple forward sine fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE SINTMF (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

FFTPACK 5.1 routine SINTMF computes the one-dimensional Fourier

transform of multiple odd sequences within a real array.

This transform is referred to as the forward transform or Fourier

analysis, transforming the sequences from physical to spectral

space.

This transform is normalized since a call to SINTMF followed

by a call to SINTMB (or vice-versa) reproduces the original

array subject to algorithmic constraints, roundoff error, etc.

Input Arguments

LOT Integer number of sequences to be transformed within

array R.

JUMP Integer increment between the locations, in array R,

of the first elements of two consecutive sequences

to be transformed.

N Integer length of each sequence to be transformed. The

transform is most efficient when N+1 is a product of

small primes.

INC Integer increment between the locations, in array R,

of two consecutive elements within the same sequence.

R Real array containing LOT sequences, each having length N.

R can have any number of dimensions, but the total number

of locations must be at least LENR.

LENR Integer dimension of R array. LENR must be at least

(LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine SINTMI before the

first call to routine SINTMF or SINTMB for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to SINTMF and SINTMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

N/2 + N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*(2*N+4).

Output Arguments

```
 R       Real output array R.  For purposes of
exposition,

         assume R's range of indices is given
by

         R(0:(LOT-1)*JUMP+(N-1)*INC).


         The output values of R are written
over the input values.

         For I=0,...,LOT-1 and J=1,...,N


          R(I*JUMP+J*INC) =


                N

                SUM
2.*R(I*JUMP+*N1*INC)*SIN(J*N1*PI/(N+1))/(N+1)

                N1=1


 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough
```

```
          =  4 input parameters INC,JUMP,N,LOT
are not consistent.



              The parameters integers INC,
JUMP, N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.



              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent ... otherwise at

              least one array element
mistakenly is transformed more

              than once.
```

COSQ1I - initialization routine for COSQ1B and COSQ1F

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```

SYNOPSIS

```
 SUBROUTINE COSQ1I (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine COSQ1I initializes
array WSAVE for use

 in its companion routines COSQ1F and COSQ1B.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate

 WSAVE arrays are required for different values
of N.


 Input Arguments


 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N is
a product of
```

```
          small primes.


 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least

          2*N + INT(LOG (REAL(N))/LOG(2.)) +4.




 Output Arguments


 WSAVE   Real work array with dimension LENSAV,
containing the

          prime factors of N and also containing
certain trigonometric

          values which will be used in routines
COSQ1B or COSQ1F.




 IER     Integer error return

          =  0 successful exit

          =  2 input parameter LENSAV not big
enough

          = 20 input error returned by lower
level routine
```

COSQ1B - real, backward quarter-cosine fast Fourier transform

```
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Programs                *
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```

SYNOPSIS

```
 SUBROUTINE COSQ1B (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine COSQ1B computes the one-
dimensional Fourier

 transform of a sequence which is a cosine
series with odd wave

 numbers.  This transform is referred to as the
backward transform

 or Fourier synthesis, transforming the
sequence from spectral to

 physical space.


 This transform is normalized since a call to
COSQ1B followed
```

by a call to COSQ1F (or vice-versa) reproduces the original

array subject to algorithmic constraints, roundoff error, etc.

Input Arguments

N Integer number of elements to be transformed in the

sequence. The transform is most efficient when N is a

product of small primes.

INC Integer increment between the locations, in array R,

of two consecutive elements within the sequence.

R Real array of length LENR containing the sequence to be

transformed.

LENR Integer dimension of R array. LENR must be at least

INC*(N-1)+ 1.

```
 WSAVE   Real work array of length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine COSQ1I before the

         first call to routine COSQ1F or COSQ1B
for a given transform

         length N.  WSAVE's contents may be re-
used for subsequent

         calls to COSQ1F and COSQ1B with the
same N.




 LENSAV  Integer dimension of WSAVE array.
LENSAV must be at least

         2*N + INT(LOG (REAL(N))/LOG(2.)) +4.




 WORK    Real array of dimension LENWRK.


 LENWRK  Integer dimension of WORK array.
LENWRK must be at

         least N.
```

Output Arguments

R Real output array. For purposes of exposition,

assume R's range of indices is given by

R(0:(N-1)*INC).

The output values of R are written over the input values.

For J=0,...,N-1

R(J*INC) =

$$\sum_{N1=0}^{N-1} R(N1*INC)*\cos(J*(2*N1+1)*PI/(2*N))$$

WSAVE Contains values initialized by subroutine COSQ1I that

must not be destroyed between calls to routine COSQ1F

or COSQ1B.

```
 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         = 20 input error returned by lower
level routine
```

COSQ1F - real, forward quarter-cosine fast Fourier transform

```
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Fast Fourier                     *
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Programs                        *
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```

SYNOPSIS

```
 SUBROUTINE COSQ1F (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine COSQ1F computes the one-
dimensional Fourier

 transform of a sequence which is a cosine
series with odd wave

 numbers.  This transform is referred to as the
forward transform

 or Fourier analysis, transforming the sequence
from physical to

 spectral space.



 This transform is normalized since a call to
COSQ1F followed

 by a call to COSQ1B (or vice-versa) reproduces
the original

 array subject to algorithmic constrain,
roundoff error, etc.



 Input Arguments



 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.
```

INC Integer increment between the locations, in array R,

of two consecutive elements within the sequence.

R Real array of length LENR containing the sequence to be

transformed.

LENR Integer dimension of R array. LENR must be at least

INC*(N-1)+ 1.

WSAVE Real work array with dimension LENSAV. WSAVE's contents

must be initialized with a call to subroutine COSQ1I before

the first call to routine COSQ1F or COSQ1B for a given

transform length N. WSAVE's contents may be re-used for

subsequent calls to COSQ1F and COSQ1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(0:(N-1)*INC).

The output values of R are written over the input values.

For J=0,...,N-1

```
          R(J*INC) =


              R(0)/N


              N-1

            + SUM
2.*R(N1*INC)*COS((2*J+1)*N1*PI/(2*N))/N

              N1=1
```

WSAVE Contains values initialized by subroutine COSQ1I that must not be destroyed between calls to routine COSQ1F or COSQ1B.

IER Integer error return

= 0 successful exit

= 1 input parameter LENR not big enough

= 2 input parameter LENSAV not big enough

= 3 input parameter LENWRK not big enough

```
          = 20 input error returned by lower
level routine
```

COSQMI - initialization routine for COSQMB and COSQMF

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SYNOPSIS

```
 SUBROUTINE COSQMI (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine COSQMI initializes
array WSAVE for use

 in its companion routines COSQMF and COSQMB.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate
```

WSAVE arrays are required for different values of N.

Input Arguments

N Integer length of each sequence to be transformed. The

transform is most efficient when N is a product of

small primes.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

Output Arguments

WSAVE Real work array with dimension LENSAV, containing the

prime factors of N and also containing certain trigonometric

values which will be used in routines COSQMB or COSQMF.

```
 IER     Integer error return

         =  0 successful exit

         =  2 input parameter LENSAV not big
enough

         = 20 input error returned by lower
level routine
```

COSQMB - real, multiple backward quarter-cosine fast Fourier transform

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SYNOPSIS

```
 SUBROUTINE COSQMB (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

FFTPACK 5.1 routine COSQMB computes the one-dimensional Fourier

transform of multiple sequences, each of which is a cosine series

with odd wave numbers. This transform is referred to as the

backward transform or Fourier synthesis, transforming the sequences

from spectral to physical space.

This transform is normalized since a call to COSQMB followed

by a call to COSQMF (or vice-versa) reproduces the original

array subject to algorithmic constraints, roundoff error, etc.

Input Arguments

LOT Integer number of sequences to be transformed within

array R.

JUMP Integer increment between the locations, in array R,

of the first elements of two consecutive sequences

to be transformed.

N Integer length of each sequence to be transformed. The

transform is most efficient when N is a product of

small primes.

INC Integer increment between the locations, in array R,

of two consecutive elements within the same sequence.

R Real array containing LOT sequences, each having length N.

R can have any number of dimensions, but the total number

of locations must be at least LENR.

LENR Integer dimension of R array. LENR must be at least

(LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array with dimension LENSAV. WSAVE's contents

must be initialized with a call to subroutine COSQMI before

the first call to routine COSQMF or COSQMB for a given

transform length N. WSAVE's contents may be re-used for

subsequent calls to COSQMF and COSQMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*N.

Output Arguments

R Real output array. For purposes of exposition,

assume R's range of indices is given by

R(0:(LOT-1)*JUMP+(N-1)*INC).

The output values of R are written over the input values.

For I=0,...,LOT-1 and J=0,...,N-1

R(I*JUMP+J*INC) =

$$\sum_{N1=0}^{N-1} R(I*JUMP+N1*INC)*\cos(J*(2*N1+1)*PI/(2*N))$$

WSAVE Contains values initialized by subroutine COSQMI that

must not be destroyed between calls to routine COSQMF

or COSQMB.

IER Integer error return

```
         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         =  4 input parameters INC,JUMP,N,LOT
are not consistent.

         = 20 input error returned by lower
level routine


              The parameters integers INC,
JUMP, N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.


              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent, otherwise at

              least one array element
mistakenly is transformed more

              than once.
```

COSQMF - real, multiple forward quarter-cosine fast Fourier transform

```
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```

SYNOPSIS

```
 SUBROUTINE COSQMF (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine COSQMF computes the one-
dimensional Fourier

 transform of multiple sequences within a real
array, where each

 of the sequences is a cosine series with odd
wave numbers.  This
```

```
 transform is referred to as the forward
transform or Fourier

 synthesis, transforming the sequences from
spectral to physical

 space.


 This transform is normalized since a call to
COSQMF followed

 by a call to COSQMB (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.


 Input Arguments


 LOT     Integer number of sequences to be
transformed within

         array R.


 JUMP    Integer increment between the
locations, in array R,

         of the first elements of two
consecutive sequences

         to be transformed.
```

```
 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
same sequence.


 R       Real array containing LOT sequences,
each having length N.

         R can have any number of dimensions,
but the total number

         of locations must be at least LENR.


 LENR    Integer dimension of R array.  LENR
must be at least

         (LOT-1)*JUMP + INC*(N-1)+ 1.




 WSAVE   Real work array o length LENSAV.
WSAVE's contents must

         be initialized with a call to
subroutine COSQMI before the
```

first call to routine COSQMF or COSQMB for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to COSQMF and COSQMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

```
          R(0:(LOT-1)*JUMP+(N-1)*INC).


          The output values of R are written
over the input values.

          For I=0,...,LOT-1 and J=0,...,N-1


           R(I*JUMP+J*INC) =


               R(I*JUMP)/N


               N-1

             + SUM
2.*R(I*JUMP+*N1*INC)*COS((2*J+1)*N1*PI/(2*N))/N

               N1=1


 IER      Integer error return

          =  0 successful exit

          =  1 input parameter LENR   not big
enough

          =  2 input parameter LENSAV not big
enough

          =  3 input parameter LENWRK not big
enough

          =  4 input parameters INC,JUMP,N,LOT
are not consistent.
```

```
         = 20 input error returned by lower
level routine



              The parameters integers INC,
JUMP, N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.



              For multiple FFTs to execute
correctly, input variables

              INC, JUMP, N and LOT must be
consistent, otherwise at

              least one array element
mistakenly is transformed more

              than once.
```

SINQ1I - initialization routine for SINQ1B and SINQ1F

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SYNOPSIS

```
 SUBROUTINE SINQ1I (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine SINQ1I initializes
array WSAVE for use

 in its companion routines SINQ1F and SINQ1B.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate

 WSAVE arrays are required for different values
of N.



 Input Arguments



 N       Integer length of the sequence to be
transformed.  The
```

transform is most efficient when N is a product of

small primes.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

Output Arguments

WSAVE Real work array with dimension LENSAV, containing the

prime factors of N and also containing certain trigonometric

values which will be used in routines SINQ1B or SINQ1F.

IER Integer error return

= 0 successful exit

= 2 input parameter LENSAV not big enough

= 20 input error returned by lower level routine

SINQ1B - real backward quarter-sine fast Fourier transform

```
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```

SYNOPSIS

```
 SUBROUTINE SINQ1B (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine SINQ1B computes the one-
dimensional Fourier

 transform of a sequence which is a sine series
with odd wave

 numbers.  This transform is referred to as the
backward transform
```

or Fourier synthesis, transforming the sequence from spectral to

physical space.

This transform is normalized since a call to SINQ1B followed

by a call to SINQ1F (or vice-versa) reproduces the original

array subject to algorithmic constraints, roundoff error, etc.

Input Arguments

N Integer length of the sequence to be transformed. The

transform is most efficient when N is a product of

small primes.

INC Integer increment between the locations, in array R,

of two consecutive elements within the sequence.

R Real array of length LENR containing the sequence to be

transformed.

LENR Integer dimension of R array. LENR must be at least

INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine SINQ1I before the

first call to routine SINQ1F or SINQ1B for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to SINQ1F and SINQ1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(INC:N*INC).

The output values of R are written over the input values.

For J=1,...,N

$$R(J*INC) = \sum_{N1=1}^{N} R(N1*INC)*SIN(J*(2*N1-1)*PI/(2*N))$$

IER Integer error return

```
          =  0 successful exit

          =  1 input parameter LENR   not big
enough

          =  2 input parameter LENSAV not big
enough

          =  3 input parameter LENWRK not big
enough

          = 20 input error returned by lower
level routine
```

SINQ1F - real forward quarter-sine fast Fourier transform

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```

SYNOPSIS

```
 SUBROUTINE SINQ1F (N, INC, R, LENR, WSAVE,
LENSAV, WORK, LENWRK, IER)

 INTEGER    N, INC, LENR, LENSAV, LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine SINQ1F computes the one-
dimensional Fourier

 transform of a sequence which is a sine series
of odd wave numbers.

 This transform is referred to as the forward
transform or Fourier

 analysis, transforming the sequence from
physical to spectral space.



 This transform is normalized since a call to
SINQ1F followed

 by a call to SINQ1B (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.



 Input Arguments



 N       Integer length of the sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.
```

INC Integer increment between the locations, in array R,

of two consecutive elements within the sequence.

R Real array of length LENR containing the sequence to be

transformed.

LENR Integer dimension of R array. LENR must be at least

INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine SINQ1I before the

first call to routine SINQ1F or SINQ1B for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to SINQ1F and SINQ1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(INC:N*INC).

The output values of R are written over the input values.

For J=1,...,N

R(J*INC) =

```
              N-1
            + SUM  (2.*R(N1*INC)*SIN(((2*J-
1)*N1*PI/(2*N)))/N
              N1=1


            + ((-1)**(J+1))*R(N*INC)/N


 IER     Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big
enough
         =  2 input parameter LENSAV not big
enough
         =  3 input parameter LENWRK not big
enough
         = 20 input error returned by lower
level routine
```

SINQMI - initialization routine for SINQMB and SINQMF

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```

SYNOPSIS

```
 SUBROUTINE SINQMI (N, WSAVE, LENSAV, IER)

 INTEGER    N, LENSAV, IER

 REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
 FFTPACK 5.1 subroutine SINQMI initializes
array WSAVE for use

 in its companion routines SINQMF and SINQMB.
The prime factor-

 ization of N together with a tabulation of the
trigonometric

 functions are computed and stored in array
WSAVE.  Separate

 WSAVE arrays are required for different values
of N.


 Input Arguments


 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N is
a product of
```

small primes.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

Output Arguments

WSAVE Real work array with dimension LENSAV, containing the

prime factors of N and also containing certain trigonometric

values which will be used in routines SINQMB or SINQMF.

IER Integer error return

= 0 successful exit

= 2 input parameter LENSAV not big enough

= 20 input error returned by lower level routine

SINQMB - real, multiple backward quarter-sine fast Fourier transform

```
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Fast Fourier                *

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Programs                *
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Dick Valent                 *
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```

SYNOPSIS

```
 SUBROUTINE SINQMB (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine SINQMB computes the one-
dimensional Fourier

 transform of multiple sequences within a real
array, where each

 of the sequences is a sine series with odd
wave numbers.  This

 transform is referred to as the backward
transform or Fourier

 synthesis, transforming the sequences from
spectral to physical
```

space.

This transform is normalized since a call to SINQMB followed

by a call to SINQMF (or vice-versa) reproduces the original

array subject to algorithmic constraints, roundoff error, etc.

Input Arguments

LOT Integer number of sequences to be transformed within

array R.

JUMP Integer increment between the locations, in array R,

of the first elements of two consecutive sequences

to be transformed.

N Integer length of each sequence to be transformed. The

transform is most efficient when N is a product of

small primes.

INC Integer increment between the locations, in array R,

of two consecutive elements within the same sequence.

R Real array containing LOT sequences, each having length N.

R can have any number of dimensions, but the total number

of locations must be at least LENR.

LENR Integer dimension of R array. LENR must be at least

(LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine SINQMI before the

first call to routine SINQMF or SINQMB for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to SINQMF and SINQMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(INC:(LOT-1)*JUMP+N*INC).

The output values of R are written over the input values.

For I=0,...,LOT-1 and J=1,...,N

R(I*JUMP+J*INC) =

$$\sum_{N1=1}^{N} R(I*JUMP+N1*INC)*SIN(J*(2*N1-1)*PI/(2*N))$$

IER Integer error return

= 0 successful exit

= 1 input parameter LENR not big enough

= 2 input parameter LENSAV not big enough

= 3 input parameter LENWRK not big enough

= 4 input parameters INC,JUMP,N,LOT are not consistent.

= 20 input error returned by lower level routine

The parameters integers INC, JUMP, N and LOT are

```
               consistent if equality

               I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

               and J1,J2 < LOT implies I1=I2 and
J1=J2.



               For multiple FFTs to execute
correctly, input variables

               INC, JUMP, N and LOT must be
consistent ... otherwise at

               least one array element
mistakenly is transformed more

               than once.
```

SINQMF - real, multiple forward quarter-sine fast Fourier transform

```
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```

SYNOPSIS

```
 SUBROUTINE SINQMF (LOT, JUMP, N, INC, R, LENR,
WSAVE, LENSAV, WORK, LENWRK, IER)
```

```
 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV,
LENWRK, IER

 REAL       R(LENR), WSAVE(LENSAV),
WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.1 routine SINQMF computes the one-
dimensional Fourier

 transform of multiple sequences within a real
array, where each

 sequence is a sine series with odd wave
numbers.  This transform

 is referred to as the forward transform or
Fourier synthesis,

 transforming the sequences from spectral to
physical space.


 This transform is normalized since a call to
SINQMF followed

 by a call to SINQMB (or vice-versa) reproduces
the original

 array subject to algorithmic constraints,
roundoff error, etc.


 Input Arguments
```

```
 LOT     Integer number of sequences to be
transformed within

         array R.


 JUMP    Integer increment between the
locations, in array R,

         of the first elements of two
consecutive sequences

         to be transformed.


 N       Integer length of each sequence to be
transformed.  The

         transform is most efficient when N is
a product of

         small primes.


 INC     Integer increment between the
locations, in array R,

         of two consecutive elements within the
same sequence.


 R       Real array containing LOT sequences,
each having length N.

         R can have any number of dimensions,
but the total number

         of locations must be at least LENR.
```

LENR Integer dimension of R array. LENR must be at least

(LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must

be initialized with a call to subroutine SINQMI before the

first call to routine SINQMF or SINQMB for a given transform

length N. WSAVE's contents may be re-used for subsequent

calls to SINQMF and SINQMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least

2*N + INT(LOG (REAL(N))/LOG(2.)) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at

least LOT*N.

Output Arguments

R Real output array R. For purposes of exposition,

assume R's range of indices is given by

R(INC:(LOT-1)*JUMP+N*INC).

The output values of R are written over the input values.

For I=0,...,LOT-1 and J=1,...,N

R(I*JUMP+J*INC) =

N-1

+ SUM (2.*R(I*JUMP+*N1*INC)*SIN(((2*J-1)*N1*PI/(2*N)))/N

N1=1

```
            + ((-1)**(J+1))*R(I*JUMP+N*INC)/N


 IER     Integer error return

         =  0 successful exit

         =  1 input parameter LENR   not big
enough

         =  2 input parameter LENSAV not big
enough

         =  3 input parameter LENWRK not big
enough

         =  4 input parameters INC,JUMP,N,LOT
are not consistent.

         = 20 input error returned by lower
level routine


              The parameters integers INC,
JUMP, N and LOT are

              consistent if equality

              I1*INC + J1*JUMP = I2*INC +
J2*JUMP for I1,I2 < N

              and J1,J2 < LOT implies I1=I2 and
J1=J2.


              For multiple FFTs to execute
correctly, input variables
```

```
              INC, JUMP, N and LOT must be
consistent ... otherwise at

              least one array element
mistakenly is transformed more

              than once.



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